TOWARDS AN “ALTERNATIVE” VIEW OF THE QUALITY OF MATHEMATICS EDUCATION IN THE SACMEQ COUNTRIES

Paper presented by
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Introduction
A great deal of educational research has shown that there is a strong linkage between the socioeconomic background of children and their educational achievement at school – for example, Coleman (1966) and Heyneman (1980, 1983). This linkage consistently demonstrates that children from higher socio-economic backgrounds tend to do better on tests of educational achievement than children from poorer backgrounds – mainly because children from wealthier homes have greater access to a range of human and material resources that encourage, facilitate, and reward school learning.

However, when discussing the performance of whole school systems, there has often been a tendency to ignore this research finding and focus instead on what are widely described as “league tables” – in which countries are ranked according to the average achievement scores of their pupils.

Comparative judgments about the performance of school systems based on this “traditional view” of school system performance are sometimes very misleading (because differences observed in average pupil achievement among school systems may be influenced by differences in pupil socio-economic intakes), and always quite narrow (because such judgments bypass important issues related to equity).

From “Traditional” to “Alternative” Views of School System Performance
Consider a hypothetical cross-national study of the quality of education in which Grade 6 pupils in countries A, B, and C are given a test that has an overall pupil average score across the three countries of 500. Also assume that the average pupil test scores for the three countries are 400, 500, and 510 respectively. The “traditional view” of school system performance would suggest that country C had the best performance, followed by country B, and then followed by country A.
Now assume that we are also provided with information that shows that the home background of pupils varies a great deal across these three countries. Country A is a moderately wealthy country where (a) people have reasonable incomes and housing; (b) the homes generally have some books, a television, and a radio; and (c) most of the parents have completed junior secondary school. Country B is a very poor country where (a) people have poverty-level incomes and housing; (b) very few homes have books or electrical appliances; and (c) many of the parents did not attend or complete primary school. Country C is a very wealthy country where (a) people have excellent incomes and housing; (b) almost all homes have many books and a wide range of electrical appliances – including computers; and (c) most parents have completed senior secondary school.

With this extra information we would probably “revise” the traditional view of school system performance as follows: First of all we would probably say that the schools in Country B (the “poor” country) had very good performance indeed because the pupils had an “average” performance of 500 despite all the hardships and disadvantages that the pupils had experienced through living in poverty with limited access to human or material educational resources in their homes.

Second, we would probably say that schools in Country A (the “reasonably wealthy” country) were exhibiting “very poor” performance (and not just “poor” performance) compared with schools in Country B – because, even though Country A pupils came from quite well resourced home backgrounds, the Country A’s average score of 400 was far lower than the average of 500 for Country B.

Third, we would probably be rather reluctant to say that the performance of the school system in Country C (the “very wealthy” country) was the best – because the gap in average mathematics scores of 10 points compared with Country B was rather small when the very wealthy home backgrounds of pupils in Country C were compared with the poverty level home backgrounds of pupils in Country B. In fact, we might even decide that the school system in Country B had the “best” performance because (after taking due consideration of the differences in pupil intakes) this school system probably increased the tested knowledge of its pupils by a much greater amount than did the school system in Country C.
It is very clear from the above discussion that the comparison of school system performance cannot simply be based on the average test scores of pupils. Rather, it is very important to judge the performance of school systems by taking account of the socioeconomic backgrounds of their pupil intakes. That is, what we really want to know when comparing the performance of school systems is “What would be the expected average test score of pupils in each country in the special (hypothetical) situation in which all school systems had pupil intakes drawn from the same socioeconomic backgrounds?” These “expected average test scores” would be more likely to provide a fairer and more valid assessment of what school systems had added to the tested knowledge of their pupil intakes.

Further, in addition to “quality”, we might also wish to know more about “equity” in school system performance – in order to check whether some school systems are better able (a) to assist disadvantaged groups (“social equity”), and (b) to ensure that both the most able pupils and less able pupils improve their knowledge (“distributional equity”).

In summary, the distortions associated with the “traditional view” of school system performance need to be adapted to form a broader “alternative view” that measures quality in a manner that adjusts for pupil intakes, and that measures the degree of social and distributional equity associated with school system performance.

**Three Dimensions of the “Alternative View” of School System Performance**

(a) **Quality**
A measure of the quality of a school system needs to be adjusted for school intakes. That is, it should provide an indication of the expected average student achievement level for a school system in a situation where the socioeconomic backgrounds of pupil intakes are equal to the average value across all school systems that are being compared. A relatively high score for a school system on this indicator would suggest that the school system would perform better than other school systems in a situation where all school systems being compared had students with the same socioeconomic background as an “average student”.

(b) **Social Equity**
A measure of social equity for a school system needs to provide an indication of the expected impact of a “unit change” in social background on pupil achievement levels. To say this in another way, the indicator should provide the expected difference in test scores between two
pupils separated by one unit on the measure of social background. If there is negligible or little difference between the expected scores of these two pupils then we can say that the system exhibits social equity. If the difference is large, then we can say that the school system has major social inequities with respect to pupil achievement.

(c) Distributional Equity

A measure of distributional equity for a school system needs to provide an indication of the “spread” in pupil achievement scores. If the spread is large then there is the possibility that while some students fare well – others will be left far behind. In fact, if the spread is very large then it is possible that a school system could have many highly successful students and many students who are complete failures!

Constructing Socioeconomic Gradient Lines

In order to apply the alternative view of school system performance in a systematic fashion to the task of comparing “real” (not hypothetical) school systems we need to have a method of quantifying the three dimensions of the alternative view of school system performance that were described above. One recent approach to this challenge has been the use of “socioeconomic gradient lines” that are based on the use of bivariate regression analysis to construct a picture or graph that summarises simultaneously all three dimensions of school system performance in a numerical form. Ross and Zuze (2004) used this approach to examine the performance of school systems in Southern and Eastern Africa with respect to reading achievement. This paper adopts the same methodology to examine performance of school systems with respect to mathematics achievement.

The measure of mathematics achievement used in this study was the numeracy test for Grade 6 pupils that was employed by the Southern Africa Consortium for Monitoring Educational Quality (SACMEQ). This study was conducted in 14 countries (Botswana, Kenya, Lesotho, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Swaziland, Tanzania (Mainland), Uganda, Zambia, and Tanzania (Zanzibar) during the period 2000-2002. (SACMEQ User’s Guide, 2004). The construction of the SACMEQ mathematics test has been described by Ratsatsi (2005), and the measure of pupil socioeconomic background was assessed by a special index of socioeconomic level that was constructed by combining variables concerned with the ‘educational level of pupils’ parents, the quality of the family
home, and a list of family possessions’. The construction of the socioeconomic index has been described in detail by Ratsatsi (2005).

<table>
<thead>
<tr>
<th>School System</th>
<th>Traditional view</th>
<th>Alternative view</th>
</tr>
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<tr>
<td>Average Pupil Mathematics Score</td>
<td>Quality Line height</td>
<td>Social Equity Line slope</td>
</tr>
<tr>
<td>Mauritius</td>
<td>585</td>
<td>474</td>
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<tr>
<td>Kenya</td>
<td>563</td>
<td>571</td>
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<tr>
<td>Seychelles</td>
<td>554</td>
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<td>Uganda</td>
<td>506</td>
<td>526</td>
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<tr>
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<td>Lesotho</td>
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<td>Malawi</td>
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<td>Namibia</td>
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</tr>
<tr>
<td>SACMEQ</td>
<td>500</td>
<td>490</td>
</tr>
</tbody>
</table>

Source: SACMEQ Data Archive (2004)

**The Main Features of the Socioeconomic Gradient Lines**

The socioeconomic gradient lines used in this study followed the Ross and Zure (2004) approach of constructing bivariate regression lines for each SACMEQ country by using pupil socioeconomic index values as the independent variable and the pupil mathematics test scores as the dependent variable. The 14 regression lines have been summarised in Table 1 and Figure 1

In the following discussion the particular technical features of the socioeconomic gradient lines have been described – in association with the conceptual “meaning” of these features. One of the most important features was concerned with moving away from “traditional” views of the quality of education based on a comparison of raw mean scores (represented by the height of the centre of each line) to an “alternative view” of the quality of education based on three components: (i) the line intercept (which portrayed the expected average pupil mathematics score if all school systems had the same average pupil socioeconomic intake), (ii) the line slope (which portrayed the degree of social equity), and (iii) the line length (which portrayed the degree of distributional equity).
Basic Characteristics of the Socioeconomic Gradient Lines

(a) The Line Variables
The equation of the regression lines took the form of:

\[ Y = aX + b \]

Where the line variables (X and Y) and were defined as follows:

\[ Y = \text{measure of pupil performance on the SACMEQ mathematics test (standardised to a mean of 500 and a standard deviation of 100).} \]

\[ X = \text{measure of pupil socioeconomic background (standardised to an average of zero and a standard deviation of 100).} \]

(b) The Line Slope (Gradient)
a = The “slope” or “gradient” of the socioeconomic gradient line. This is equal to the increase in pupil mathematics performance (Y) for each standard deviation (100 units) increase in pupil socioeconomic level (X). For clarity of presentation in Table 1, the slope values have been multiplied by 100.

The slopes of the socioeconomic gradient lines represent the degree of social equity in pupil mathematics achievement. These values quantify the impact of a one standard deviation change (100 units) in pupil socioeconomic background on pupil mathematics achievement. It may be seen in Table 1 that the average line slope for all SACMEQ school systems was 32, and the school system line slopes ranged from a low of 6 for Mozambique to a high of 96 for Mauritius.

Smaller line slopes (as for Mozambique) implied greater social equity. Steeper line slopes (as from Mauritius) suggested large differences in average pupil mathematics achievement across different socioeconomic groups – with the potential danger that pupils from poorer backgrounds might be left far behind pupils from wealthier backgrounds.

The lowest levels of social equity occurred for Mauritius, Seychelles, and South Africa where the lines slopes were 96, 56, and 54 respectively. The highest levels of social equity occurred for Mozambique, Zanzibar, and Lesotho where the line slopes were around 6 to 8.
(c) Line Intercept

\[ b = \text{The “intercept” of the socioeconomic gradient line. That is, the “height” at which the line crossed the Y axis. From Figure 1 it may be seen that this is equal to the value of expected pupil mathematics achievement (Y) when the pupils socioeconomic level (X) is equal to zero (which represents the average socioeconomic level).} \]

The heights of the socioeconomic gradient lines represented adjusted quality measures. These values estimated the expected average pupil mathematics achievement when all school systems had pupil socioeconomic intakes that were equal to the SACMEQ average. In Table 1 it may be seen that the average line height for all SACMEQ school systems was 490 points, and the school system line heights ranged from a low of 437 points for Namibia to a high of 571 points for Kenya.

The line heights (illustrated in Figure 1 by the heights of the line intercepts with the vertical axis) provided a “fairer” and more meaningful alternative approach to comparing the contributions of school systems to the quality of educational outputs – because these figures had a built-in adjustment for different pupil socioeconomic intakes.

Two Other Important Characteristics of Socioeconomic Gradient Lines

(a) Line Length

The length of the socioeconomic gradient lines gave an indication of the “distance” in test score points between lower achieving pupils and higher achieving pupils in each country. This measure represented the degree of distributional equity in pupil reading achievement.

The line lengths were scaled to be equal to the variance (divided by 100) of the pupil mathematics scores. The average line length for all SACMEQ school systems was 77, and the school system line lengths ranged from low values of around 30 in Malawi and Mozambique, to a very high value of 196 in Mauritius.

Shorter line lengths implied greater distributional equity. Greater line lengths suggested major differences between the mathematics achievements of the most able and least able pupils – with the potential danger that some less able pupils could be left far behind.
The lowest levels of distributional equity occurred for Mauritius, South Africa, Uganda and Seychelles – where the variances all exceeded the SACMEQ average by 40 points. The higher levels of distributional equity occurred for Malawi, Mozambique, and Lesotho – where the variances were less than half the SACMEQ average.

(b) Line Centre
An important property of a regression line using two variables, X and Y, is that it always passes through the point associated with the means of both variables. This point is often referred to as the “centre” of the line. In Figure 1 these points have been marked on each of the regression lines prepared for the SACMEQ countries. The vertical Y axis refers to pupil mathematics scores, and therefore the height of the line centre of the line above the horizontal X axis is equal to the average pupil score on the mathematics test (Y).

Discussion of Socio-Economic Gradient Lines
The SACMEQ countries have been ranked in Table 1 according to the heights of the line centres. The heights of the centres of the socioeconomic gradient lines above the horizontal axis provided a traditional “league table” perspective of quality - because these values were equal to the average student mathematics score for each country.

From a traditional perspective, the “best” school systems were Mauritius, Kenya, and the Seychelles – with average pupil mathematics scores from around 55 to 85 score points above the SACMEQ average of 500. The school systems rating lowest from the traditional perspective were Namibia, Malawi, and Zambia – all being around 65 to 70 score points below the SACMEQ average.

The “alternative” perspective of school system performance used in this study borrows its operational definition from the approach used by Ross and Zuze (2004) to study reading achievement in the SACMEQ studies. This perspective moves away from the simplistic league table approach towards a broader view of quality that considers the performance of school systems as a combination of three benchmarks. The Ross and Zuze (2004) operational definition was adapted to focus on mathematics achievement in the following fashion.

High performance school systems should demonstrate:
• high quality – illustrated by high values on an indicator of expected average pupil mathematics achievement in a situation where the socioeconomic backgrounds of pupil intakes are equal to the average across school systems; and

• high social equity – illustrated by low values on an indicator of the impact of pupil socio-economic background on mathematics achievement; and

• high distributional equity – illustrated by low values on an indicator on the spread in pupil mathematics achievement.

(a) Adjusted quality
It is very interesting to contrast the traditional and alternative perspectives on “quality” which are presented in the first two columns of figures in Table 1. For example, the best school system from a traditional perspective was Mauritius with an average student mathematics score of 585. However, the expected student mathematics score for Mauritius – under the assumption that the socioeconomic background of its student intake was equal to the SACMEQ average – was only around 474. This result suggested that the high average test scores obtained by Mauritius students were substantially influenced by the fact that the students were drawn from much wealthier socioeconomic backgrounds than the SACMEQ average.

The reverse effect occurred for Tanzania and Uganda – where the expected average student mathematics scores under the alternative perspective were (a) around 20 test score points higher than the traditional average student mathematics scores, and (b) higher than the expected student mathematics score for Mauritius. In other words, the expected average student mathematics achievement of students in Tanzania and Uganda was higher than for Mauritius – after making a statistical adjustment for the differences in student intakes across the countries.

(b) Social Equity
Mauritius had the steepest line slope at 96 points while the lowest line slope occurred for Mozambique and Zanzibar, where the slopes were 6 points. The SACMEQ average line slope was 32 points. These results showed that in Mauritius there was very high social inequity
when compared to other SACMEQ countries. To explain this in another way, it can be said that in Mauritius there was an expected 96 point difference in achievement scores for pupils who differed by 100 units in their socioeconomic background. The line slope for Mauritius was 64 points above the SACMEQ average and 40 points above Seychelles – the next highest country. Another country with steep slopes indicating social inequity was South Africa with a slope of 54 points.

Other countries however had socioeconomic gradient lines with gentle line slopes – which indicated greater social equity. These countries were Mozambique, Zanzibar, and Lesotho. In these three countries we would expect to see a difference of less than ten test score points between two pupils who differed in socioeconomic background by 100 score points.

Countries that had line slopes which were less than the average SACMEQ line slope of 32 were considered to have greater social equity, while those with higher line slopes had greater social inequity.

(c) Distributional Equity
In this analysis, distributional equity is measured using the length of the socioeconomic gradient lines. Mauritius (196) had the longest line length, while Malawi (32) and Mozambique (32) had the shortest line lengths. The average SACMEQ distributional equity as measured by line length was 77. Mauritius has a line length of 119 above the SACMEQ average implying that, while there were many pupils who were successful in their mathematics examinations, there were also many pupils who were doing poorly. That is, distributional equity was a major problem in Mauritius when compared to other SACMEQ countries.

In contrast, Malawi and Mozambique had the shortest line lengths implying that pupils in these countries were achieving at a more similar level. That is there was a very small difference between the high achievers and the low achievers.

Countries which were below the SACMEQ average line length were Malawi, Mozambique, Lesotho, Zanzibar, Swaziland, Zambia, Botswana, Namibia, and Tanzania. These countries could be said to have some distributional equity. However in countries such as Zambia,
Malawi, and Namibia it should also be said that the pupils were performing at an equally poor level.

**Towards an Overall Conclusion About School System Performance**

An examination of Figure 1 provides an interesting way of quickly determining the performance of the SACMEQ school systems. The “best” school system has a socioeconomic gradient line that is short, flat, and has a high intercept with the vertical axis.

Given this perspective, the lines in Figure 1 indicated, for example, that Mozambique was doing very well on social equity (flat line) and distributional equity (short line), whereas Mauritius was not doing well on social equity (steep line) or distributional equity (long line). In contrast, Malawi, Zambia, Namibia, and (surprisingly) Mauritius were all doing poorly on adjusted quality (low intercept).

How can these disparate features be somehow summarised and combined so that we can decide which school systems have the best performance from an “alternative” point of view? This question has been addressed in the following discussion.

**Which School Systems are Best?**

In this study “high” and “low” values on the three alternative dimensions were derived with reference to the SACMEQ average values – and indicated in bold when these values were “better” than the SACMEQ average value. That is, a high performance school system was defined as one that satisfied all three of the following benchmarks: High quality – with line heights greater than the SACMEQ average of 490; High social equity – with line slopes less than the SACMEQ average of 32; and High distributional equity – with line lengths less than the SACMEQ average of 77. The figures in bold in the final three columns of Table 1 designated benchmarks that were satisfied by school systems.

The three school systems that delivered the highest quality mathematics education from the “traditional” point of view were those at the top of Table 1 with the highest average student mathematics achievement. However, **none** of these school systems satisfied all three of the high performance bench marks. Kenya satisfied the quality and the distributional equity benchmarks – but not the “social equity” benchmark. Mauritius and Seychelles did **not** satisfy any of the three benchmarks. So, from an alternative point of view, the school systems in
Mauritius, Kenya, and the Seychelles could not be considered in the high performance category.

Only three school systems satisfied all three of the alternative perspectives benchmarks for high performance: Mozambique, Swaziland and Botswana. These school systems were “better” than the SACMEQ averages related to quality, social equity, and distributional equity.

**Conclusion**
The ranking of schools in “league tables” using average test scores is a misleading approach to the comparison of school system performance. The reason for this is that this approach does not take into consideration the differences between school systems with respect to their pupil socioeconomic intakes.

The alternative view however accounts for the socioeconomic background of pupil intakes. The alternative view looks at three dimensions of (a) quality – which is adjusted for school system intakes; (b) social equity – the expected impact of a unit change in social background on pupil achievement levels; and (c) distributional equity – the spread in pupil achievement scores.

The analysis presented in this paper showed that the three SACMEQ school systems (Mauritius, Kenya, and the Seychelles) that were the best performers for a “traditional view” did not satisfy all three conditions that had been specified for high performance under the “alternative view”.

Only three SACMEQ school systems could be said to be performing well in the area of mathematics. These were the countries that satisfied all three alternative requirements: Mozambique, Swaziland and Botswana.
References


Figure 1: Socioeconomic Gradient Lines - SACMEQ II (Mathematics)